# Protein Secondary Structure Prediction Using Dynamic Programming 

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#### Abstract

In the present paper, we describe how a directed graph was constructed and then searched for the optimum path using a dynamic programming approach, based on the secondary structure propensity of the protein short sequence derived from a training data set. The protein secondary structure was thus predicted in this way. The average three-state accuracy of the algorithm used was $76.70 \%$.


Key words directed graph; dynamic programming approach; protein secondary structure

Protein structure prediction helps to facilitate our understanding of the protein function. It is commonly recognized that the 3-D structure of a protein can be accurately predicted when the prediction accuracy of the secondary structure reaches $80.00 \%$. The prediction of the secondary structure using the primary structure is the main obstacle when predicting the 3-D structure of a protein. When predicting the secondary structure, the three-state accuracy $Q_{3}$ is used as a criterion to assess the prediction accuracy,
$Q_{3}=\frac{N_{\alpha}+N_{\beta}+N_{\mathrm{c}}}{N}$
where $N_{\alpha}, N_{\beta}$ and $N_{c}$ are respectively the number of residues in $\alpha$-helix, $\beta$-sheet and other types predicted correctly, and $N$ is the total number of amino acid residues predicted.

A number of computational methods have been developed for predicting the protein secondary structure, such as information theory methods, the nearest-neighbor method and the artificial neural network method. Information theory methods are based on the statistical characteristics of a single amino acid's propensity for a given

[^0]conformational state. Examples of such methods include GOR1 [1], GOR3 [2], GORIV [3], and DSC [4]. Their $Q_{3}$ values are about 69.50\%. Zvelebil et al. [5] used the alignment of homologous sequences and got a $Q_{3}$ value of $66.00 \%$. The nearest-neighbor method is based on the conformational states of the best matches or nearest neighbors. An example of such a method is PREDATOR [6], whose $Q_{3}$ is about $68.00 \%$. Yi and Lander's algorithm [8], NNSSP [9] and PHD [10] are based on artificial neural networks. The highest $Q_{3}$ of these artificial neural network methods is $74.00 \%$. Recently, Ward et al. [11] used support vector machines and got a $Q_{3}$ value of $77.07 \%$. Recent researches in this area have been mainly focused on the incorporation of existing methods to improve the prediction accuracy.

In the present paper, we introduce a novel method for the secondary structure prediction of a protein.

## Analysis of Short Peptide Propensity

First, we downloaded all the 24,310 protein sequences and their secondary structure parameters from the DSSP (Database of Secondary Structure in Proteins, http://www. sander.ebi.ac.uk/dssp/) and NLR-3D [the Sequence-structure Database produced from sequence and annotation information extracted from three-dimensional structures in the Protein Databank (PDB), http://pir.georgetown.edu/ pirwww/dbinfo/nrl3d.html]. Then we deleted the redundant and inferior sequences according to the following rules: (1) omit the homologous sequences; (2) delete those sequences with the wrong secondary structure notation; and (3) delete those sequences designated to be of low
quality by PROCHECK which checks the stereochemical quality of a protein structure (http://www.biochem.ucl. ac.uk/~roman/procheck/procheck.html) [6]. This process resulted in a set of 5100 sequences with high quality annotation in the secondary structure that was used for the short peptide propensity analysis.

The short peptides were divided into 10 secondary structure classes: (1) connecting peptides between $\alpha$-helix and $\alpha$-helix, denoted by $\alpha \alpha$; (2) connecting peptides between $\alpha$-helix and $\beta$-sheet, denoted by $\alpha \beta$; (3) connecting peptides between $\beta$-sheet and $\beta$-sheet, denoted by $\beta \beta$; (4) connecting peptides between $\beta$-sheet and $\alpha$-helix, denoted by $\beta \alpha$; (5) beginning peptides of $\alpha$-helix, denoted by $\alpha_{B}$; (6) beginning peptides of $\beta$-sheet, denoted by $\beta_{\mathrm{B}}$; (7) terminal peptides of $\alpha$-helix, denoted by $\alpha_{\mathrm{E}}$; (8) terminal peptides of $\beta$-sheet, denoted by $\beta_{\mathrm{E}}$; (9) $\alpha$-helical peptides, denoted by $\alpha$; and (10) $\beta$-sheet peptides, denoted by $\beta$.

All the peptides of the 5100 sequences make up set $\Omega$. Those peptides belonging to $\alpha \alpha, \alpha \beta, \beta \beta, \beta \alpha, \alpha_{B}, \beta_{B}, \alpha_{E}$, $\beta_{\mathrm{E}}$, $\alpha$ and $\beta$, respectively, comprise subset $\Omega_{i}, i=0,1, \cdots, 9$. The number of peptides in each secondary structure class is listed in Table 1.

Let $N\left(w, \Omega_{i}\right)$ be the occurrence frequency of peptide $w$ in set $\Omega_{i}$. The secondary structure propensity coefficient (SSPC) $P\left(w, \Omega_{i}\right)$ is then defined by,
$P\left(w, \Omega_{i}\right)=\frac{N\left(w, \Omega_{i}\right)}{N(w, \Omega)}, i=0,1, \cdots, 9$
The peptide conflict rate is defined as the percentage of peptides that belong to two or more secondary structure classes ( $\alpha \alpha, \alpha \beta, \beta \beta, \beta \alpha, \alpha_{B}, \beta_{B}, \alpha_{E}, \beta_{E}, \alpha, \beta$ ) in the total number of peptides. Through statistical analysis, we found that when the length of the peptide $L$ is 4 amino acids, the conflict rate is too high and the secondary structure propensity is too low to be used in the prediction of the secondary structure. Statistical analysis results also show that $L=5$ amino acids is the best peptide length for the propensity analysis.

## Construction of the Directed Graph

For a protein sequence $X=x_{0} x_{1} \cdots x_{N-1}$, where $N$ is the
length of the sequence, the short peptide $w[j]$ of length $L$ for position $j$ is defined as follows:
$w[j]=x_{j} X_{j+1} \cdots x_{j+L-1} \quad j=0,1, \cdots, N-L$
The SSPC of $w[j]$ is denoted by $P\left(w[j], \Omega_{i}\right)(i=0,1, \cdots, 9)$. The SSPCs of sequence $X$ make up the matrix $P(X)$ of $10 \times(N-L+1)$ :
$P(X)=\left[\begin{array}{cccc}P\left(w[0], \Omega_{0}\right) & P\left(w[1], \Omega_{0}\right) & \ldots & P\left(w[N-L], \Omega_{0}\right) \\ P\left(w[0], \Omega_{1}\right) & P\left(w[1], \Omega_{1}\right) & \ldots & P\left(w[N-L], \Omega_{1}\right) \\ \vdots & \vdots & \vdots & \vdots \\ P\left(w[0], \Omega_{9}\right) & P\left(w[1], \Omega_{9}\right) & \ldots & P\left(w[N-L], \Omega_{9}\right)\end{array}\right]$
When the propensity coefficient of $\alpha \alpha P\left(w[j], \Omega_{0}\right)>0$, the short peptide in position $j$ is probably a connecting peptide of $\alpha \alpha$. This $\alpha \alpha$ peptide is equivalent to a terminal peptide of $\alpha_{E}$ in position $j-1$ and a beginning peptide of $\alpha_{\mathrm{B}}$ in position $j+1$. So the propensity coefficients of $\alpha_{\mathrm{E}}$ in $j-1$ and $\alpha_{\text {B }}$ in $j+1$ are both equal to $P\left(w[j], \Omega_{0}\right)$. Therefore, for simplification, the propensity coefficients of $\alpha \alpha$ can be included in the propensity coefficients of $\alpha_{B}$ and $\alpha_{E}$ by modifying the propensity coefficients of $\alpha_{E}$ and $\alpha_{\mathrm{B}}$ as follows:

$$
\begin{aligned}
& P\left(w[j-1], \Omega_{6}\right)=\max \left\{P\left(w[j], \Omega_{0}\right), P\left(w[j-1], \Omega_{6}\right)\right\} \\
& P\left(w[j+1], \Omega_{4}\right)=\max \left\{P\left(w[j], \Omega_{0}\right), P\left(w[j+1], \Omega_{4}\right)\right\}
\end{aligned}
$$

Similarly, the propensity coefficients of $\alpha \beta$ can be included in the propensity coefficients of $\alpha_{E}$ and $\beta_{\mathrm{B}}$ by modifying the propensity coefficients of $\alpha_{E}$ and $\beta_{B}$ as follows:

$$
\begin{aligned}
& P\left(w[j-1], \Omega_{6}\right)=\max \left\{P\left(w[j], \Omega_{1}\right), P\left(w[j-1], \Omega_{6}\right)\right\} \\
& P\left(w[j+1], \Omega_{5}\right)=\max \left\{P\left(w[j], \Omega_{1}\right), P\left(w[j+1], \Omega_{5}\right)\right\}
\end{aligned}
$$

The propensity coefficients of $\beta \beta$ can be included in the propensity coefficients of $\beta_{\mathrm{E}}$ and $\beta_{\mathrm{B}}$ by modifying the propensity coefficients of $\alpha_{E}$ and $\beta_{\mathrm{B}}$ as follows:

$$
\begin{aligned}
& P\left(w[j-1], \Omega_{7}\right)=\max \left\{P\left(w[j], \Omega_{2}\right), P\left(w[j-1], \Omega_{7}\right)\right\} \\
& P\left(w[j+1], \Omega_{5}\right)=\max \left\{P\left(w[j], \Omega_{2}\right), P\left(w[j+1], \Omega_{5}\right)\right\}
\end{aligned}
$$

The propensity coefficients of $\beta \alpha$ can be included in the propensity coefficients of $\beta_{\mathrm{E}}$ and $\alpha_{\mathrm{B}}$ by modifying the propensity coefficients of $\alpha_{\mathrm{E}}$ and $\beta_{\mathrm{B}}$ as follows:
$P\left(w[j-1], \Omega_{7}\right)=\max \left\{P\left(w[j], \Omega_{3}\right), P\left(w[j-1], \Omega_{7}\right)\right\}$
$P\left(w[j+1], \Omega_{4}\right)=\max \left\{P\left(w[j], \Omega_{3}\right), P\left(w[j+1], \Omega_{4}\right)\right\}$

Table 1 The number of peptides in each secondary structure class

| $\Omega_{0}(\alpha \alpha)$ | $\Omega_{1}(\alpha \beta)$ | $\Omega_{2}(\beta \beta)$ | $\Omega_{3}(\beta \alpha)$ | $\Omega_{4}\left(\alpha_{\mathrm{B}}\right)$ | $\Omega_{5}\left(\beta_{\mathrm{B}}\right)$ | $\Omega_{6}\left(\alpha_{\mathrm{E}}\right)$ | $\Omega_{7}\left(\beta_{\mathrm{E}}\right)$ | $\Omega_{8}(\alpha)$ | $\Omega_{9}(\beta)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10,785 | 15,247 | 16,397 | 15,950 | 28,843 | 35,409 | 28,802 | 35,423 | 797,536 | 282,152 |

The propensity coefficient $P\left(w[j], \Omega_{4}\right)$ of $\alpha_{\mathrm{B}}$ in position $j$ means that the probability of occurrence of the secondary structure $\alpha_{\mathrm{B}}$ in position $j$ is $P\left(w[j], \Omega_{4}\right)$. In addition, when the propensity coefficients of the $\alpha$ peptide in position $j, j+1$ and $j+2$ are high, the credibility of the secondary structure $\alpha_{\mathrm{B}}$ in position $j$ increases; otherwise, it decreases. Therefore, the credibility of $\alpha_{B}$ is defined as:

$$
\begin{aligned}
S\left(j, \alpha_{\mathrm{B}}\right) & =P\left(w[j], \Omega_{4}\right)\left[1+\left(P\left(w[j], \Omega_{8}\right)+P\left(w[j+1], \Omega_{8}\right)\right.\right. \\
& \left.\left.+P\left(w[j+2], \Omega_{8}\right)\right) / 3\right]
\end{aligned}
$$

The credibility of $\beta_{\mathrm{B}}$ is defined in the same way:

$$
\begin{aligned}
S\left(j, \beta_{\mathrm{B}}\right) & =P\left(w[j], \Omega_{5}\right)\left[1+\left(P\left(w[j], \Omega_{9}\right)+P\left(w[j+1], \Omega_{9}\right)\right.\right. \\
& \left.\left.+P\left(w[j+2], \Omega_{9}\right)\right) / 3\right]
\end{aligned}
$$

The propensity coefficient $P\left(w[j], \Omega_{6}\right)$ of $\alpha_{\mathrm{E}}$ in position $j$ means that the probability of occurrence of the secondary structure $\alpha_{\mathrm{E}}$ in position $j$ is $P\left(w[j], \Omega_{6}\right)$. Additionally, when the propensity coefficients of the $\alpha$ peptide in positions $j, j-1$ and $j-2$ are high, the credibility of the secondary structure $\alpha_{\mathrm{E}}$ in position $j$ increases; otherwise, it decreases. Therefore, the credibility of $\alpha_{E}$ is defined as:

$$
\begin{aligned}
S\left(j, \alpha_{\mathrm{E}}\right) & =P\left(w[j], \Omega_{6}\right)\left[1+\left(P\left(w[j], \Omega_{8}\right)+P\left(w[j-1], \Omega_{8}\right)\right.\right. \\
& \left.\left.+P\left(w[j-2], \Omega_{8}\right)\right) / 3\right]
\end{aligned}
$$

and the credibility of $\beta_{\mathrm{E}}$ is defined similarly as:

$$
\begin{aligned}
S\left(j, \beta_{\mathrm{E}}\right) & =P\left(w[j], \Omega_{7}\right)\left[1+\left(P\left(w[j], \Omega_{9}\right)+P\left(w[j-1], \Omega_{9}\right)\right.\right. \\
& \left.\left.+P\left(w[j-2], \Omega_{9}\right)\right) / 3\right]
\end{aligned}
$$

Therefore, the protein sequence $X=x_{0} x_{1} \cdots x_{N-1}$ corresponds to a matrix $S(X)$ :
$S(X)=\left[\begin{array}{cccc}S\left(0, \alpha_{\mathrm{B}}\right) & S\left(1, \alpha_{\mathrm{B}}\right) & \ldots & S\left(N-L, \alpha_{\mathrm{B}}\right) \\ S\left(0, \beta_{\mathrm{B}}\right) & S\left(1, \beta_{\mathrm{B}}\right) & \ldots & S\left(N-L, \beta_{\mathrm{B}}\right) \\ S\left(0, \alpha_{\mathrm{E}}\right) & S\left(1, \alpha_{\mathrm{E}}\right) & \ldots & S\left(N-L, \alpha_{\mathrm{E}}\right) \\ S\left(0, \beta_{\mathrm{E}}\right) & S\left(1, \beta_{\mathrm{E}}\right) & \ldots & S\left(N-L, \beta_{\mathrm{E}}\right)\end{array}\right]$
Finally, a directed graph $G$ is constructed from $S(X)$ as follows.
(1) The vertex set $\{\operatorname{node}(j), j=1,2, \cdots, k\}$ is composed of $k$ vertices.

A vertex in a directed graph $G$ is defined as the linking region between two secondary structures. Its data structure is:
node $(j)$ \{float $\alpha_{E}$ score, $\beta_{E}$ score, $\alpha_{B}$ score, $\beta_{B}$ score; int $\alpha_{E}$ position, $\beta_{E}$ position, $\alpha_{B}$ position, $\beta_{B}$ position, position $\}$
where $\alpha_{E}$ score, $\beta_{E}$ score, $\alpha_{B}$ score and $\beta_{B}$ score are the respective credibility scores of $\alpha_{\mathrm{E}}, \beta_{\mathrm{E}}, \alpha_{\mathrm{B}}$ and $\beta_{\mathrm{B}}$ of node(j); $\alpha_{E}$ position, $\beta_{E}$ position, $\alpha_{B}$ position and $\beta_{B}$ position are the respective positions of $\alpha_{E}, \beta_{E}, \alpha_{B}$ and $\beta_{B}$; and position is
the position of the vertex node $(j)$ in $X$. If there are $k$ vertices in $S(X)$, then the parameters of $\operatorname{node}(j), j=1,2, \cdots, k$ are calculated as follows:
node $(j) . \alpha_{E} \operatorname{score}=S\left(i_{1}{ }^{*}, \alpha_{E}\right) ;$ node $(j) . \alpha_{E}$ position $=i_{1}{ }^{*}$
where $i_{1}{ }^{*}$ satisfies:
$S\left(i_{1}{ }^{*}, \alpha_{E}\right)=\max \left\{S\left(i, \alpha_{E}\right), \operatorname{position}[j-1]<i<\operatorname{position}[j]\right\}$ node $(j) . \beta_{E} \operatorname{score}=S\left(i_{2}{ }^{*}, \beta_{E}\right) ; \operatorname{node}(j) . \beta_{E}$ position $=i_{2}{ }^{*}$
where $i_{2}{ }^{*}$ satisfies:
$S\left(i_{2}^{*}, \beta_{E}\right)=\max \left\{S\left(i, \beta_{E}\right)\right.$,position $\left.[j-1]<i<\operatorname{position}[j]\right\}$
node $(j) . \alpha_{B}$ score $=S\left(i_{3}{ }^{*}, \alpha_{B}\right) ;$ node $(j) . \alpha_{B}$ position $=i_{3}{ }^{*}$
where $i_{3}{ }^{*}$ satisfies:
$S\left(i_{3}{ }^{*}, \alpha_{\mathrm{B}}\right)=\max \left\{S\left(i, \alpha_{B}\right)\right.$, position $\left.[j]<i<\operatorname{position}[j+1]\right\}$ $\operatorname{node}(j) . \beta_{\mathrm{B}} \operatorname{score}=S\left(i_{4}{ }^{*}, \beta_{B}\right) ; \operatorname{node}(j) . \beta_{B}$ position=$i_{4}{ }^{*}$
where $i_{4}{ }^{*}$ satisfies:
$S\left(i_{4}^{*}, \beta_{\mathrm{B}}\right)=\max \left\{S\left(i, \beta_{B}\right)\right.$, position $[j]<i<$ position $\left.[j+1]\right\}$
where $j=1,2, \cdots, k$, and assuming position $[0]=0$, position $[k+1]=N-L$.
(2) The weights of the directed arc from node(i) to node(j) represent the secondary structure propensity from node( $i$ ) to node $(j)$ in $G$, where $i<j$. These are defined respectively as:

$$
\begin{align*}
& \omega_{\alpha}(i, j)=\frac{\sqrt{\text { node }(i) \cdot \alpha_{\mathrm{B}} \operatorname{score} \times \operatorname{node}(j) \cdot \alpha_{\mathrm{E}} \operatorname{score}}}{j-i}, \\
& 1 \leq i<j \leq k  \tag{5}\\
& \omega_{\beta}(i, j)=\frac{\sqrt{\text { node }(i) \cdot \beta_{\mathrm{B}} \operatorname{score} \times \text { node }(j) \cdot \beta_{\mathrm{E}} \operatorname{score}}}{j-i}
\end{align*}
$$

$$
\begin{equation*}
1 \leq i<j \leq k \tag{6}
\end{equation*}
$$

where the denominator $j-i$ represents the penalty for the leaping over the vertices between node( $j$ ) and node( $i$ ).

From the definitions above, the graph $G$ with $k$ nodes for secondary structure prediction is presented as the following matrix:

$$
G=\left[\begin{array}{cccccc}
0 & \omega_{\alpha}(1,2) & \omega_{\alpha}(1,3) & \omega_{\alpha}(1,4) & \cdots & \omega_{\alpha}(1, k) \\
\omega_{\beta}(1,2) & 0 & \omega_{\alpha}(2,3) & \omega_{\alpha}(2,4) & \cdots & \omega_{\alpha}(2, k) \\
\omega_{\beta}(1,3) & \omega_{\beta}(2,3) & 0 & \omega_{\alpha}(3,4) & \cdots & \omega_{\alpha}(3, k) \\
\omega_{\beta}(1,4) & \omega_{\beta}(2,4) & \omega_{\beta}(3,4) & 0 & \cdots & \omega_{\alpha}(4, k) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\omega_{\beta}(1, k) & \omega_{\beta}(2, k) & \omega_{\beta}(3, k) & \omega_{\beta}(4, k) & \cdots & 0
\end{array}\right]
$$

Every element (from the second element onward) in the first row represents the weight of the $\alpha$ structure from the first vertex to other vertices, and every element (from the second element onward) in the first column represents the weight of the $\beta$ structure from the first vertex to other vertices. Similarly, every element (from the third element onward) in the second row represents the weight of the $\alpha$ structure from the second vertex to other vertices, and every element (from the third element downward) in the second column represents the weight of the $\beta$ structure from the second vertex to other vertices. The rest of the elements can be similarly explained. For example, a directed graph $G$ with four vertices is shown in Fig. 1.


Fig. 1 The directed graph with four vertices for the protein secondary structure prediction

## Searching for the Optimum Path

Utilizing the directed graph $G$ with $k$ vertices defined above, the secondary structure of the corresponding protein can be predicted. In graph $G$, a single directed path from the initial vertex to the terminal vertex represents one solution of the secondary structure prediction.

Let $E_{x}\left(i_{p}, i_{p+1}\right)$ represent the directed arc from $\operatorname{node}\left(i_{p}\right)$ to $\operatorname{node}\left(i_{p+1}\right)$ connected by structure $x(x=\alpha$ or $\beta)$. The symbol " $\succ$ " is used to denote the junction between two arcs. Therefore, Path $(i, j)$, which is the path composed of $z$ arcs from node( $i$ ) to node $(j)$, is denoted as:

$$
\begin{aligned}
& \operatorname{Path}(i, j)=E_{x}\left(i, i_{1}\right) \succ E_{x}\left(i_{1}, i_{2}\right) \succ E_{x}\left(i_{2}, i_{3}\right) \succ \cdots \succ E_{x}\left(i_{z-1}, j\right), \\
& i<i_{1}<i_{2}<i_{3}<\cdots<i_{z-1}<i_{z}=j
\end{aligned}
$$

The weight of the path is defined as:
$\omega(\operatorname{Path}(i, j))=\left[\omega_{x}\left(i, i_{1}\right)+\omega_{x}\left(i_{1}, i_{2}\right)+\cdots+\omega_{x}\left(i_{z-1}, j\right)\right] / z$
This means the weight of $\operatorname{Path}(i, j)$ is defined as the
average weights of all the directed arcs along this path. From this definition, the optimum path from the initial vertex to the terminal vertex of graph $G$-namely the path with the highest $w(\operatorname{Path}(1, k))$-represents the optimum solution of the secondary structure prediction, corresponding to the solution with the highest mean credibility.

For graph $G$ with $k$ vertices, there exist $2^{k} k$ ! paths from the initial vertex to the terminal vertex. The task of the protein secondary structure prediction becomes transformed into one of finding an optimum path, namely Path ${ }^{*}$, which maximizes the $w\left(\right.$ Path $\left.^{*}\right)$ in the $2^{k} k$ ! paths. This task has exponential computational complexity. Such a challenge can be overcome efficiently by a dynamic programming approach (DPA).

Computing the optimum path by a dynamic programming approach is based on the optimum principle:

If $\operatorname{Path}^{*}(1, i)$ is the optimum path from the initial vertex node(1) to the $i$ th vertex of graph $G$ and $E^{*}(i, j)$ is the optimum directed arc from node(i) to node $(j)$, with $j \in\{i+1$, $i+2, \cdots, k\}$, then
$\operatorname{Path}^{*}(1, j)=$ Path $^{*}(1, i) \succ E^{*}(i, j)$
is the optimum path from the initial vertex node(1) to node (j).

Based on this principle, when computing the optimum path from the initial vertex node(1) to node(j), the optimum paths from node(1) to the senior vertices of node ( $j$ ) should be computed in advance. Therefore, we may begin from the initial vertex node(1), and compute the optimum path from senior vertices to junior vertices.

Three parameters of node $(j)$ are defined as follows:
$v(j)$ : score of $\operatorname{node}(j), v(j)=\max \{\omega(\operatorname{Path}(1, j))\}$, representing the weight of the optimum path from the initial vertex to node(j);
$b(j)$ : number of arcs in the optimum path from the initial vertex node(1) to node(j);
$U(j)$ : ordered array composed of the vertex code and structure types, representing the optimum path from node(1) to node $(j)$, where $j=1,2, \cdots, k$, and $v(1)=0, b(1)=0, U(1)$ $=\{1\}$.

Two parameters for the directed arc from node(i) to node $(j)$ are defined as follows:
$\omega_{x}(i, j)$ : weights of the optimum directed arc from node( $(i)$ to $\operatorname{node}(j) ; \omega_{x}(i, j)=\max \left\{\omega_{\alpha}(i, j), \omega_{\beta}(i, j)\right\}$;
$B(i, j)$ : structure types of the optimum directed arc from node $(i)$ to $\operatorname{node}(j)$; if $\omega_{x}(i, j)=\omega_{\alpha}(i, j)$, then $B(i, j)=\alpha$, otherwise $B(i, j)=\beta$, where $1 \leq i<j \leq k$.

The dynamic programming algorithm that searches for the optimum path is described in detail as follows:
Step 1: for $1 \leq i<j \leq k$, calculate $\omega_{x}(i, j)$ and $B(i, j)$ :
$\omega_{\chi}(i, j)=\max \left\{\omega_{\alpha}(i, j), \omega_{\beta}(i, j)\right\}$
if $\omega_{x}(i, j)=\omega_{\alpha}(i, j)$, then
$B(i, j)=\alpha$, otherwise $B(i, j)=\beta$.
Step 2: let $v(1)=0, b(1)=0, U(1)=\{1\}$;
Step 3: for each $j=2,3, \cdots, k$, compute $v(j), b(j)$ and $U(j)$ successively:
$v(j)=\max \left\{\left.\frac{v(p) \times b(p)+\omega_{x}(p, j)}{b(p)+1} \right\rvert\, p=1,2, \cdots, j-1\right\}$
(1) If there exists only one $p^{*}$ such that
$v(j)=\frac{v\left(p^{*}\right) \times b\left(p^{*}\right)+\omega_{x}\left(p^{*}, j\right)}{b\left(p^{*}\right)+1}$
then
$b(j)=b\left(p^{*}\right)+1$
Add $B\left(p^{*}, j\right)$ and $j$ orderly to the end of set $U\left(p^{*}\right)$, and obtain $U(j)$.
(2) If there exist $p_{1}, p_{2}, \cdots, p_{k}$, such that $1 \leq p_{1}<p_{2}<\cdots$ $<p_{k} \leq j-1$, and
$\frac{v\left(p_{1}\right) \times b\left(p_{1}\right)+\omega_{x}\left(p_{1}, j\right)}{b\left(p_{1}\right)+1}=\cdots=\frac{v\left(p_{k}\right) \times b\left(p_{k}\right)+\omega_{x}\left(p_{k}, j\right)}{b\left(p_{k}\right)+1}$
$=v(j)$
then
$b(j)=b\left(p_{k}\right)+1$
Add $B\left(p_{k}, j\right)$ and $j$ orderly to the end of set $U\left(p_{2}\right)$, and obtain $U(j)$ (the weight of the vicinity vertex is preferential).

Finally, we obtain $v(k)$, which is the weight of the optimum path from the initial vertex to the terminal vertex, and $U(k)$, which is the corresponding trial optimum path.

For example, the optimum path in Fig. 2 is 1- $\beta-2-\alpha-3-\alpha-4$, which means the $\beta$ structure is predicted from node(1) to node (2), and the $\alpha$ structure is predicted from node(2) to node (4).


Fig. 2 Weighted direction graph with four nodes for protein secondary structure prediction

## Results and Discussion

The data from 130 low-homologous proteins selected by Rost and Sander [10] were used to test the algorithm described here.

In Test 1, we divided the data into four groups, namely $\alpha, \beta, \alpha+\beta \& \alpha / \beta$ and others, and predicted the protein secondary structure with GOR3, PHD and the DPA introduced in this paper. The $Q_{3}$ values of these methods are compared in Table 2. We can see that DPA performed better than GOR3 and PHD in almost every case in Test 1.

Test 2 was carried out to further investigate the performance of DPA. In Test 2, we partitioned the sequences into two subsets, Set I and Set II. Set I contains those sequences of which more than $90 \%$ are longer than 5 amino acids, while Set II contains the remaining sequences. The prediction results are listed in Table 3.

From Table 2, we can see that the performance of DPA

Table $2 \quad Q_{3}$ comparison of the prediction results in Test 1

| Case | Sequence number | Residue number | GOR3 $Q_{3}(\%)$ | PHD $Q_{3}(\%)$ | DPA $Q_{3}(\%)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| All $\alpha$ | 23 | 3247 | 64.7 | 83.10 | 80.53 |
| All $\beta$ | 10 | 1092 | 48.6 | 73.96 | 75.60 |
| $\alpha+\beta \& \alpha / \beta$ | 40 | 9955 | 57.9 | 76.15 | 76.46 |
| Others | 10,143 | 57.7 | 72.67 | 75.93 |  |
| Total | 57 | 24,437 | 58.3 | 75.69 | 76.70 |

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Table $3 \quad Q_{3}$ comparison of the prediction results in Test 2

| Set | Sequence number | Residue number | GOR3 $Q_{3}(\%)$ | PHD $Q_{3}(\%)$ | DPA $Q_{3}(\%)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| I | 28 | 3434 | 65.25 | 83.90 | 96.53 |
| II | 102 | 21,003 | 57.15 | 73.51 | 73.45 |
| Total | 130 | 24,437 | 58.30 | 75.69 | 76.70 |

for Set I is much better than that of GOR3 and PHD, while the results for Set II show no significant difference between DPA and PHD. This is because DPA's SSPC database omits those peptides whose lengths are less than 5 amino acids. Fortunately, Set II only contains a small number of sequences.

## Conclusion

DPA can overcome the shortcomings of the methods based on a single amino acid's propensity because it utilizes SSPC, and it is faster because of the dynamic programming algorithm. DPA will perform even better if combined with other methods.

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